Report 3

Digital Signal Processing

Solution of Q1

Given $x(n) = \cos(\omega n)\, u(n)$

Find $X(z)$

Sol^n

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-k}$$

$$\because \cos(\omega n) = \frac{e^{j\omega n} + e^{-j\omega n}}{2} = x(n)$$

$$\therefore X(z) = \sum_{k=0}^{\infty} \frac{e^{j\omega k} + e^{-j\omega k}}{2} z^{-k}$$

$$= \frac{1}{2}\left[\sum_{k=0}^{\infty} e^{j\omega k} z^{-k} + \sum_{k=0}^{\infty} e^{-j\omega k} z^{-1} \right]$$

$$= \frac{1}{2}\left[1 + e^{j\omega} z^{-1} + e^{j\omega \times 2} z^{-2} + \cdots + 1 + e^{-j\omega} z^{-1} + e^{-j2\omega} z^{-2} + \cdots \right]$$

$$= \frac{1}{2}\left[1 + e^{j\omega} z^{-1} + (e^{j\omega} z^{-1})^2 + \cdots + 1 + e^{-j\omega} z^{-1} + (e^{-j\omega} z^{-1})^2 + \cdots \right]$$

hint, $1 + x + x^2 + \cdots = \frac{1}{1-x}$, $x \ll 1$

$$\therefore X(z) = \frac{1}{2}\left[\frac{1}{1 - e^{j\omega} z^{-1}} + \frac{1}{1 - e^{-j\omega} z^{-1}} \right]$$

$$= \frac{1}{2}\left[\frac{1 - e^{-j\omega} z^{-1} + 1 - e^{j\omega} z^{-1}}{(1 - e^{j\omega} z^{-1})(1 - e^{-j\omega} z^{-1})} \right]$$

$$= \frac{1}{2}\left[\frac{2 - (e^{j\omega} + e^{-j\omega}) z^{-1}}{1 - (e^{j\omega} + e^{-j\omega}) z^{-1} + z^{-2}} \right]$$

To be Continued

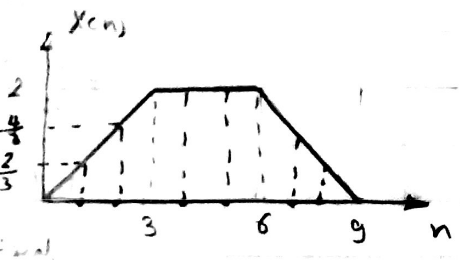
$$X(z) = \frac{1 - \left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) z^{-1}}{1 - 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) z^{-1} + z^{-2}}$$

$$= \frac{1 - \cos(\omega)\, z^{-1}}{1 - 2z^{-1}\cos\omega + z^{-2}} = \frac{z^2 - \cos(\omega)\, z}{z^2 - 2z\cos\omega + 1}$$

$$\therefore \boxed{X(z) = \frac{z(z - \cos(\omega))}{z^2 - 2z\cos(\omega) + 1}}$$

Solution of Q2:

For the given signal $x(n)$
Find $X(z)$.



Sol^n

$$x(n) = \left\{0, \tfrac{2}{3}, \tfrac{4}{3}, 2, 2, 2, 2, \tfrac{4}{3}, \tfrac{2}{3}, 0\right\}$$

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-1} = x(0) + x(1) z^{-1} + x(2) z^{-2} + \dots$$

$$= 0 \cdot 1 + \frac{2}{3} z^{-1} + \frac{4}{3} z^{-2} + 2(z^{-3} + z^{-4} + z^{-5} + z^{-6}) + \frac{4}{3} z^{-7} + \frac{2}{3} z^{-8}$$

$$\boxed{X(z) = \frac{2}{3}(z^{-1} + z^{-8}) + \frac{4}{3}(z^{-2} + z^{-7}) + 2(z^{-3} + z^{-4} + z^{-5} + z^{-6})}$$